REVIEWS

Jets, Wakes and Cavities, by GARRETT BIRKHOFF and E. H. ZARANTONELLO. New York : Academic Press, 1957. 353 pp. $10.00 or 80*s*.

There can be little doubt that Helmholtz, with his paper " Über discontinuerliche Flüssigkeitsbewegungen " (1868), laid one of the cornerstones of modern fluid mechanics. He himself, with the editors of *The Philosophical Magazine*, who published a translation of the paper in the same year, was confident of its importance—just as, thirty-six years later, Prandtl was confident in his paper on the boundary layer. Yet, Helmholtz seems not to have known which cornerstone he was laying, and probably he imagined that he was laying Prandtl's.

Certainly the discontinuous solutions of Laplace's equation constructed by Helmholtz, and by his successors such as Kirchhoff (who introduced the name ' free streamline ') and Rayleigh, have found effective application almost exclusively to flows of liquid bordered by regions of gas or vapour—notably, liquid jets in a gaseous atmosphere, and liquid flows about obstacles with cavity formation—whereas the flows of a single homogeneous fluid, to which the authors originally intended the solutions to apply, have proved too complicated for detailed study by these means. Meanwhile, the theory and practice of constructing such discontinuous solutions have been developed to a high degree of virtuosity by an army of workers, and many interesting phenomena have been illuminated as a result. Nevertheless, too few of those writing on the subject have grasped its true relation to physical phenomena, and many have repeated almost word for word the errors of the founders of the theory, such as Helmholtz's ascription of flow separation from a solid surface to causes which really are those underlying cavitation. Especial harm has been done by the writers of hydrodynamical textbooks, who have actively propagated false views of the significance of free-streamline theory.

The vast structure built upon Helmholtz's original paper—comprising the mathematical theory of the existence and uniqueness of flows with free streamlines ; the techniques of obtaining analytical expressions for such flows in particular cases and then translating them into numerical results ; the use of the solutions to throw light on jet collisions, on planing, on the effectiveness of shaped charges, on vapour-filled and air-filled cavities, on contraction coefficients and flows over weirs and waterfalls ; the experimental and theoretical study of the physical conditions in flows involving jets and cavities and the relation between these conditions and aspects of the mathematical solutions ; the further study of possible relationships between separated flows of a homogeneous fluid and particular types of free-streamline solution—all this is a structure spanning the widely different disciplines of pure and applied mathematics, and involving substantial parts of physics and engineering. The need for a unified account of all this varied material, which would help those working on isolated parts of it to see the material as a whole, and reduce the danger of their misinterpreting its significance, has long existed, as has the difficulty of finding anyone competent to undertake such a synthesis.

Now Professors Garrett Birkhoff and E. H. Zarantonello, both distinguished workers in the field, have collaborated to produce just such a unified account, and it is a real pleasure to report that they have succeeded, achieving both the necessary breadth and the necessary depth. Professor Birkhoff is to be congratulated on having made the difficult transition from pure to applied mathematics, at least for the purpose of writing parts of this book; and those applied mathematicians who found their enjoyment of his *Hydrodynamics* spoilt by its oversimplified view of the relations between natural science and mathematics can be assured that this experience will not be repeated.

The authors are particularly strong adherents to the view that free streamlines are valuable only as representations of liquid–gas boundaries. With the principal object of emphasizing this, they include three chapters on wakes and jets in the flow of homogeneous fluid (for example, air jets in air), chapters which summarize rather briefly the knowledge in these fields but at least make clear to any doubters that an approach to them by means of free-streamline solutions must be quite inadequate. With the exception of these chapters (12–14) the book is a study of cavities and liquid jets, and of the theory of the mathematical determination of free-streamline solutions.

The authors are careful not to give an oversimplified view of the cavitation phenomenon, and they devote their final chapter to its many complexities as well as to descriptions of the various instabilities which lead to the break-up of jets.

Naturally, one of the most useful features of the book is the 'zoo' of basic free-streamline flows, that is, two-dimensional irrotational flows with either the magnitude or the direction of the velocity uniform along each portion of the boundary. The zoo is well arranged, in three chapters (2, 3 and 5) dealing with types of flow in increasing order of complexity. In these chapters there are some sixty sketches of particular configurations, showing simply the solid boundaries and the free streamlines in each case, which, with a sentence or two in the text, identify the flow clearly. In addition, the analytical expressions for the solution are given, and remarks about its utility, if any. In chapter 9 the numerical computation of these flows is systematically discussed. Streamline shapes and other details of the internal flow are obtained in many cases and exhibited in a series of plates at the back of the book. This is a big advance, since, in previous work, details only of the boundary streamlines had been computed for flows of this kind. In addition, many tables and graphs are given of the relations between the defining parameters for particular types of flow.

The general theory of the existence, uniqueness and nature of free-streamline flows, including those with curved solid boundaries, is treated in chapters 4, 6 and 7. A knowledge of at least the results of this material is of the greatest help to those using free-streamline solutions. Readers wishing to understand the proofs will have to make a very real effort, but fortunately they are set out in a civilized way which does not prevent a reader skimming the cream of the results while only sipping at the difficult ideas underlying their demonstration.

The first half of chapter 8 is devoted to the Chaplygin compressible-flow free-streamline theory of gaseous jets, a subject included somewhat inconsistently in view of the authors' general contention that only liquid jets are well represented by free-streamline theory. This chapter ends with some remarks about the problem of including gravitational effects. However, it gives no account of perturbation methods in this connection, and computations by the relaxation method are not mentioned, being only briefly treated in a section at the end of chapter 9, whereas they have proved especially fruitful for calculating free-streamline flows under gravity. Chapter 10 is a good account of what is known about axisymmetric free-streamline flows, and chapter 11 takes us rapidly, but with references, through an extensive miscellany of unsteady flows with free boundaries.

When one looks back over this book, which is such a welcome addition to the shelf of really good hydrodynamical treatises, it is difficult to find any general questions on which one would like to challenge the authors. However, I will try. Perhaps the one possible fault is an inevitable consequence of one of the book's chief virtues, namely its insistence (so necessary in view of the errors of the past) that flows of homogeneous fluid with separation are too complicated to be represented by free-streamline solutions. At the end of the book, on the other hand, one would like to ask: "Given that this is indisputably true in the main, are there not still occasional points where free-streamline theory can throw some light on flows of homogeneous fluid?" In other words, was Helmholtz completely wrong?

For example, the authors' figure 3 in chapter 2 indicates that the pressure distribution on a flat plate, at an angle of incidence of 15° or more, is close to that given by one of the cavitation models (such as that with a re-entrant jet, but admittedly all the models give much the same distribution over the plate), with a cavity pressure equal to that found experimentally on the leeward side of the plate. Also, figures 2 in both chapter 1 and chapter 14 indicate that a re-entrant jet model is a reasonable qualitative representation of the flow near the plate, although the real flow becomes fuzzier and fuzzier (relative to the model) as one goes away from the plate.

Again, there are experimental indications that vortex sheets in the neighbourhood of solid boundaries are fairly stable, so that free-streamline models may be more useful for representing 'separation bubbles' (that is, finite dead-air regions produced by boundary-layer separation followed by reattachment) than, say, wakes. The choice of model for a separation bubble would involve boundary-layer considerations to determine the position of separation.

But it probably needs to be pointed out that the authors of this book are at their weakest in boundary-layer considerations. Fortunately, the main sections of the book do not lose much from this, since on most solid boundaries in the free-streamline solutions which are discussed the flow is accelerating, so that boundary layers should be thin and should not separate.

However, there are several flows illustrated involving flow round a corner in a wall (see chapter 2, figures 4a, 4b, 6d ; chapter 3, figures 10b, 12a ; chapter 5, figures 7a, 7b, 8a, 8b, 8c, 8d, 8e, 10a, 10b, 10c). In all of these, boundary-layer separation would cause large inaccuracies in the solution—a circumstance to which the reader's attention should have been drawn more explicitly.

Actually, there is no evidence in the book itself that the authors are aware of this fact in the case of corners concave to the flow. If they had been, they would presumably not have delayed the only discussion of boundary layers and separation until chapter 14, where a single page is devoted to the subject. This page, indeed, is a peculiar one, apparently designed to play down the role of the boundary layer, and it fails to state clearly that separation will not occur at all in flows with sufficiently small adverse pressure gradients along solid boundaries.

It is surprising that authors who have written at such length on irrotational flows should wish to play down the boundary layer, by whose presence alone irrotational flows are made possible in the face of the hydrodynamic boundary condition of zero relative velocity at a solid boundary. The Navier–Stokes equations for incompressible flow are exactly satisfied by irrotational flow fields, but these are compatible with the boundary condition only if a boundary layer intervenes between them and any solid boundary. A flow irrotational outside this thin layer will be possible only if the equations governing the dynamics of the layer, in the environment of the given irrotational flow, permit a steady solution without such a singularity as is implied by the separation condition. Divergences between theory and experiment for boundary layers which do separate (that is, for which the equations reach this singularity) constitute no criticism of a theory whose most important conclusion is the condition under which separation will be absent.

However, this condition is not satisfied by any flow along a wall up to a stagnation point (like a concave corner). It is for this reason that flows involving separation bubbles (see for example chapter 5, figures 2b, 2c, 2d in this book) have been proposed to describe such cases. The present reviewer has argued for such a model in the case of flow up a step (apparently without convincing Professor Birkhoff, for one cannot, alas, suppose the reference unknown to such an accomplished bibliographer !). In addition, there is reason to hope that free-streamline theories of separation bubbles on thin aerofoils at high angle of incidence may gradually achieve greater accuracy.

Still, it must be confessed that the authors' case remains a tenable one, and that Helmholtz's vindication is yet to come. In the meantime, the appearance of this powerful critical survey of the present position in the whole subject may be regarded as bearing witness to the vitality of his original conception. Our thanks are due to the authors for making such a book as pleasant to read as it is comprehensive and instructive.

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